

筋上有开口裂缝的加筋矩形板横向振动特性分析

王志强, 周 叮, 霍瑞丽, 李雪红

(南京工业大学土木工程学院, 江苏 南京 211816)

摘要: 利用能量法分析筋上含有开口裂缝的四边简支加筋矩形板的横向振动。将矩形板与加强筋沿交界面切开, 再将加强筋沿裂缝分成多个子块, 采用薄板弯曲和平面应力理论分别建立矩形板和各子块的横向振动能量方程, 解决了传统分析方法需给定开裂处筋的弯曲刚度问题。采用第一类切比雪夫多项式构造矩形板和各子块的位移试函数, 由 Ritz 法和板-筋界面变形连续条件得出含有开口裂缝的加筋矩形板的横向振动特征方程。计算结果与有限元分析结果吻合很好, 详细分析了裂缝深度和裂缝位置对无量纲频率的影响。

关键词: 加筋矩形板; 横向振动; 裂缝; Chebyshev-Ritz 法

中图分类号: TU311.3 **文献标志码:** A **文章编号:** 1004-4523(2023)04-0946-07

DOI: 10.16385/j.cnki.issn.1004-4523.2023.04.007

引 言

加筋板的稳定性和承载力较大, 因而在土木、机械等工程中被广泛应用, 例如跨度较大的加筋楼板、带肋的桥面板等结构, 由于结构初始缺陷和长期承受动荷载作用, 加强筋容易产生裂缝。近年来国内外对加筋板结构振动特性的分析很多, 但是对于含有开口裂缝的加筋矩形板振动特性的研究还相对较少。因此需要了解此类型加筋矩形板的振动特性, 为其在工程应用中提供一定的理论依据。

对于加筋板振动特性的分析, 通常采用有限元等数值计算或者试验测试。Rao 等^[1]考虑了加筋板内加强筋分布角度的影响, 将加筋板离散化, 利用 Lagrange 方程求解加筋板的固有频率。Samanta 等^[2]提出了一种通用的三节点三角形壳单元, 可以灵活地分析加筋板的振动特性。Peng 等^[3]基于一阶剪切变形理论提出了无网格伽辽金方法。Bhaskar 等^[4]使用弹性力学和经典力学计算了四边简支加筋板的无量纲频率, 并与各种近似模型进行了比较。李凯等^[5]利用能量泛函变分以及拉格朗日乘子得到加筋板的特征方程。杜菲等^[6]利用 Rayleigh-Ritz 法求解四边固支加筋板的固有频率, 计算结果与 ANSYS 分析和实验结果吻合均较好。游翔宇等^[7]运用边光滑有限元方法对加筋板的静

力问题和动力问题进行分析。石鹏^[8]建立了曲型加筋板的分析模型, 使用较少网格即可得到较为准确的结果。裂缝梁模型主要分为张开裂缝模型^[9]和呼吸裂缝模型^[10-11]。Lee^[12]用边界元法计算裂缝梁的固有频率。蒋杰等^[13-14]利用能量法和 Ritz 法分析裂缝梁的自由振动。孔成等^[15]将裂缝梁分为两段进行研究, 给出了悬臂裂缝梁的特征方程, 进而得到了梁的固有频率。Xue 等^[16]研究带有 I 型裂纹的矩形板在平行于板边的一侧加筋后的自由振动问题。Peng 等^[17]利用一阶剪切变形理论(FSDT), 讨论了加强筋对有裂纹的矩形板的位移、应力和应力强度因子的影响。

在大多数情况下, 传统方法都是使用扭簧近似裂缝处梁的弯曲刚度, 梁的固有频率有无穷多个, 因此扭簧刚度和裂缝的关系只能依据基阶频率近似给出。本文利用 Chebyshev-Ritz 建立矩形板以及各个子块的特征方程, 引入位移连续条件得到含有开口裂缝的加筋矩形板的振动特征方程。该方法无需使用裂缝等效刚度, 具有简单直接的特点。

1 加筋矩形板横向振动分析模型

1.1 含有开口裂缝的加筋矩形板模型

图 1 为一加筋矩形板, 矩形板下表面有单根带裂缝加强筋。其中矩形板的长度, 宽度和厚度分别为 a , b 和 h 。加强筋的宽度和高度分别为 B 和 H 。

收稿日期: 2021-11-27; **修订日期:** 2022-02-15

基金项目: 国家自然科学基金资助项目(51778285, 52178486); 江苏省交通运输科技项目(2021Y17-2); 江苏省研究生科研与实践创新计划项目(KYCX21_1147)。

加强筋的裂缝考虑为开口型,即裂缝在荷载作用下不会闭合。矩形板的密度为 ρ ,泊松比为 μ ,弹性模量为 E 。如无特别说明,加强筋与矩形板的材料相同。建立如图1所示坐标系,加强筋在 $y=0$ 处,矩形板和加强筋任一点沿 z 方向的位移为 $w(x,y,t)$,加强筋任一点沿 x 方向的位移为 $u(x,y,t)$ 。

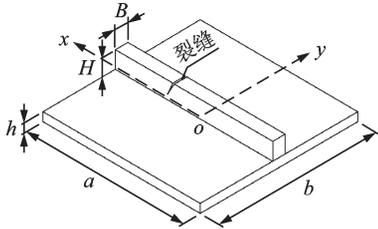


图1 含有开口裂缝的加筋矩形板模型

Fig. 1 Model of stiffened rectangular plate with open crack

1.2 矩形板的横向振动分析

由于矩形板的平面尺寸比厚度大得多,可用薄板理论进行分析,从而忽略水平方向的惯性力,只考虑板的横向惯性力。

基于薄板理论,矩形板横向振动的应变能和动能^[18]分别为:

$$U_p = \frac{D}{2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} [(\frac{\partial^2 w}{\partial x^2})^2 + (\frac{\partial^2 w}{\partial y^2})^2 + 2\mu \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + 2(1-\mu)(\frac{\partial^2 w}{\partial x \partial y})^2] dx dy \quad (1)$$

$$T_p = \frac{\rho h}{2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} (\frac{\partial w}{\partial t})^2 dx dy \quad (2)$$

式中 $D = Eh^3/[12(1-\mu^2)]$ 为板的弯曲刚度。

不计阻尼,自由振动时矩形板 z 方向的位移函数分量为:

$$w(x,y,t) = W(x,y)e^{i\omega t} \quad (3)$$

式中 ω 为矩形板横向振动的特征频率; $W(x,y)$ 为板的振型; $i = \sqrt{-1}$; t 为时间。

将 x,y 坐标无量纲化,即:

$$\xi_p = 2x/a, \eta_p = 2y/b \quad (4)$$

将式(3)和(4)代入式(1)和(2),可得矩形板最大应变能和动能分别为:

$$U_{\max,p} = \frac{2bD}{a^3} \int_{-1}^1 \int_{-1}^1 [(\frac{\partial^2 W}{\partial \xi_p^2})^2 + \lambda^4 (\frac{\partial^2 W}{\partial \eta_p^2})^2 + 2\lambda^2 \mu \frac{\partial^2 W}{\partial \xi_p^2} \frac{\partial^2 W}{\partial \eta_p^2} + 2\lambda^2(1-\mu)(\frac{\partial^2 W}{\partial \xi_p \partial \eta_p})^2] d\xi_p d\eta_p \quad (5)$$

$$T_{\max,p} = \frac{ab\rho h}{8} \omega^2 \int_{-1}^1 \int_{-1}^1 W^2 d\xi_p d\eta_p \quad (6)$$

式中 $\lambda = a/b$ 为板的长宽比。

用第一类切比雪夫多项式构造矩形板横向振动

的位移试函数为:

$$W(\xi_p, \eta_p) = f_{w,1}(\xi_p) f_{w,2}(\eta_p) \sum_{i=1}^{I_p} \sum_{j=1}^{J_p} A_{ij}^p P_i(\xi_p) P_j(\eta_p) \quad (7)$$

式中 A_{ij}^p 为位移函数中相应项的未知系数; I_p 和 J_p 为切比雪夫级数项; $P_s(x)$ ($x = \xi_p, \eta_p$; $s = i, j$) 为一维 s 阶第一类切比雪夫多项式; $f_{w,1}(\xi_p)$ 和 $f_{w,2}(\eta_p)$ 分别为 x 和 y 方向满足结构边界条件的边界函数分量。

本文研究的矩形板为四边简支(SSSS),边界函数分量为:

$$f_{w,1}(\xi_p) = 1 - \xi_p^2, f_{w,2}(\eta_p) = 1 - \eta_p^2 \quad (8)$$

其他边界条件也可以建立类似的边界函数。

$P_s(x)$ ($x = \xi_p, \eta_p$) 用余弦函数形式表示为:

$$P_s(x) = \cos[(s-1)\arccos x] \quad s = 1, 2, 3, \dots \quad (9)$$

1.3 加强筋的横向振动分析

如图2所示,设裂缝至加强筋左端和右端的距离分别为 a_2 和 a_3 ,裂缝深度为 H_2 ,裂缝尖端至梁上表面的距离为 H_1 。将加强筋分成三个子块,分别建立三个局部坐标系。

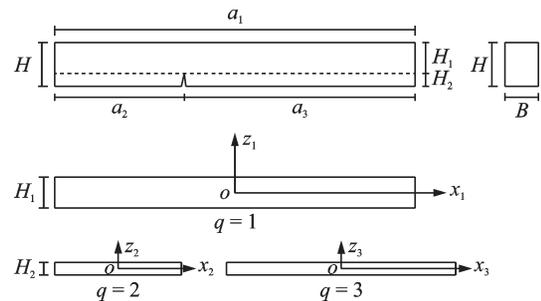


图2 加强筋模型

Fig. 2 Stiffener model

采用平面应力理论,每个子块($q=1,2,3$)的应变能和动能^[19]分别为:

$$U_q = \frac{BE}{2(1-\mu^2)} \int_{-\frac{a_q}{2}}^{\frac{a_q}{2}} \int_{-\frac{H_q}{2}}^{\frac{H_q}{2}} [(\frac{\partial u_q}{\partial x})^2 + 2\mu \frac{\partial u_q}{\partial x} \frac{\partial w_q}{\partial z} + (\frac{\partial w_q}{\partial z})^2 + \frac{1-\mu}{2} (\frac{\partial u_q}{\partial z} + \frac{\partial w_q}{\partial x})^2] dx dz \quad (10)$$

$$T_q = \frac{\rho_q B}{2} \int_{-\frac{a_q}{2}}^{\frac{a_q}{2}} \int_{-\frac{H_q}{2}}^{\frac{H_q}{2}} [(\frac{\partial u_q}{\partial t})^2 + (\frac{\partial w_q}{\partial t})^2] dx dz \quad (11)$$

式中 u_q 和 w_q 分别为各子块在 x 和 z 方向的位移。令 $H_3 = H_2$,将 x_q 和 z_q ($q=1,2,3$) 无量纲化,即:

$$\xi_q = \frac{2x_q}{a_q}, \zeta_q = \frac{2z_q}{H_q} \quad (12)$$

自由振动时各子块在 x 和 z 方向的位移分别为:

$$\begin{aligned} u_q(\xi_q, \zeta_q, t) &= U_q(\xi_q, \zeta_q)e^{i\omega t}, \\ w_q(\xi_q, \zeta_q, t) &= W_q(\xi_q, \zeta_q)e^{i\omega t} \end{aligned} \quad (13)$$

将式(12), (13)代入式(10), (11)中, 可得各子块的最大应变能和动能分别为:

$$\begin{aligned} U_{\max, q} &= \frac{EB}{2\theta_q(1-\mu^2)} \int_{-1}^1 \int_{-1}^1 \left[(\theta_q \frac{\partial U_q}{\partial \xi_q})^2 + (\frac{\partial W_q}{\partial \zeta_q})^2 + \right. \\ &\quad \left. 2\mu\theta_q \frac{\partial U_q}{\partial \xi_q} \frac{\partial W_q}{\partial \zeta_q} + \frac{1-\mu}{2} (\frac{\partial U_q}{\partial \zeta_q} + \theta_q \frac{\partial W_q}{\partial \xi_q})^2 \right] d\xi_q d\zeta_q \end{aligned} \quad (14)$$

$$T_{\max, q} = \frac{\rho_q B a_q H_q \omega^2}{8} \int_{-1}^1 \int_{-1}^1 (U_q^2 + W_q^2) d\xi_q d\zeta_q \quad (15)$$

式中 $\theta_q = \frac{H_q}{a_q}$.

用第一类切比雪夫多项式构造各子块的位移函数为:

$$\begin{aligned} U_q(\xi_q, \zeta_q) &= \phi_u^q(\xi_q) \varphi_u^q(\zeta_q) \sum_{m=1}^{M_q} \sum_{n=1}^{N_q} B_{mn}^q P_m(\xi_q) P_n(\zeta_q), \\ W_q(\xi_q, \zeta_q) &= \phi_w^q(\xi_q) \varphi_w^q(\zeta_q) \sum_{l=1}^{L_q} \sum_{r=1}^{R_q} C_{lr}^q P_l(\xi_q) P_r(\zeta_q) \end{aligned} \quad (16)$$

式中 B_{mn}^q 和 C_{lr}^q 为位移函数中相应项的未知系数; M_q, N_q, L_q 和 R_q 为切比雪夫级数项; $\phi_u^q(\xi_q), \varphi_u^q(\zeta_q)$ 和 $\phi_w^q(\xi_q), \varphi_w^q(\zeta_q)$ 分别为各子块在 x 和 z 方向的边界函数分量。

本文加强筋两端为简支, 三个子块的边界函数分别为:

$$\begin{aligned} \phi_u^1(\xi_1) &= 1, \phi_w^1(\xi_1) = 1 - \xi_1^2, \phi_u^2(\xi_2) = \phi_w^2(\xi_2) = 1, \\ \phi_u^3(\xi_3) &= 1, \phi_w^3(\xi_3) = 1 - \xi_3^2, \phi_u^4(\xi_4) = \phi_w^4(\xi_4) = 1 \end{aligned} \quad (17)$$

其他边界条件也可以建立类似的边界函数。

1.4 含有开口裂缝的加筋矩形板的横向振动分析

含有开口裂缝的加筋矩形板的最大应变能和动能分别为:

$$\begin{aligned} U_{\max} &= U_{\max, p} + \sum_{q=1}^3 U_{\max, q}, \\ T_{\max} &= T_{\max, p} + \sum_{q=1}^3 T_{\max, q} \end{aligned} \quad (18)$$

定义加筋矩形板的能量泛函为:

$$\Pi = U_{\max} - T_{\max} \quad (19)$$

由 Rayleigh-Ritz 法, 得:

$$\frac{\partial \Pi}{\partial A_{ij}^p} = 0, \frac{\partial \Pi}{\partial B_{mn}^q} = 0, \frac{\partial \Pi}{\partial C_{lr}^q} = 0 \quad (20)$$

将式(5), (6), (14)和(15)代入式(19)和(20), 得到矩形板和各子块横向振动的特征方程为:

$$\left(\frac{4bD}{a^3} K_p - \frac{ab\rho h\omega^2}{4} M_p \right) X_p = 0,$$

$$\left(\frac{EB}{2\theta_q(1-\mu^2)} K_q - \frac{\rho B a H \beta_q \gamma_q \omega^2}{4} M_q \right) X_q = 0 \quad (21)$$

式中

$$\begin{aligned} K_q &= \begin{pmatrix} K_{uu, q} & K_{uw, q} \\ K_{wu, q} & K_{ww, q} \end{pmatrix}, M_q = \begin{pmatrix} M_{uu, q} & \\ & M_{ww, q} \end{pmatrix}, \\ X_p &= A_p = \{A_{11, p} \ A_{12, p} \cdots A_{21, p} \ A_{22, p} \cdots A_{i1, p} \cdots A_{ij, p}\}^T, \\ X_q &= \{B_q^T \ C_q^T\}^T, \\ B_q &= \{B_{11, q} \ B_{12, q} \cdots B_{21, q} \ B_{22, q} \cdots B_{m1, q} \cdots B_{mn, q}\}^T, \\ C_q &= \{C_{11, q} \ C_{12, q} \cdots C_{21, q} \ C_{22, q} \cdots C_{l1, q} \cdots C_{lr, q}\}^T \end{aligned} \quad (22)$$

式中 K_p, K_q 和 M_p, M_q 分别为矩形板和各子块的刚度矩阵和质量矩阵; X_p 和 X_q 分别为未知系数 A_{ij}^p 和 B_{mn}^q, C_{lr}^q 构成的列向量。 $\beta_q = a_q/a, \gamma_q = H_q/H$ 。其中

$$\begin{aligned} K_p &= [F_{w\bar{w}i\bar{i}}^{(2,2)} G_{w\bar{w}j\bar{j}}^{(0,0)} + \lambda^4 F_{w\bar{w}i\bar{i}}^{(0,0)} G_{w\bar{w}j\bar{j}}^{(2,2)} + \lambda^2 \mu (F_{w\bar{w}i\bar{i}}^{(0,2)} G_{w\bar{w}j\bar{j}}^{(2,0)} + \\ &\quad F_{w\bar{w}i\bar{i}}^{(2,0)} G_{w\bar{w}j\bar{j}}^{(0,2)}) + 2\lambda^2 (1-\mu) F_{w\bar{w}i\bar{i}}^{(1,1)} G_{w\bar{w}j\bar{j}}^{(1,1)}], \\ K_{uu, q} &= [2\theta_q^2 D_{u\bar{u}m\bar{m}}^{(1,1)} Q_{u\bar{u}n\bar{n}}^{(0,0)} + (1-\mu) D_{u\bar{u}m\bar{m}}^{(0,0)} Q_{u\bar{u}n\bar{n}}^{(1,1)}], \\ K_{uw, q} &= [2\mu\theta_q D_{u\bar{u}m\bar{m}}^{(1,0)} Q_{u\bar{u}n\bar{n}}^{(0,1)} + (1-\mu)\theta_q D_{u\bar{u}m\bar{m}}^{(0,1)} Q_{u\bar{u}n\bar{n}}^{(1,0)}], \\ K_{ww, q} &= [2D_{w\bar{w}l\bar{l}}^{(0,0)} Q_{w\bar{w}r\bar{r}}^{(1,1)} + (1-\mu)\theta_q^2 D_{w\bar{w}l\bar{l}}^{(1,1)} Q_{w\bar{w}r\bar{r}}^{(0,0)}], \\ K_{uw, q} &= K_{uw, q}^T, M_p = F_{w\bar{w}i\bar{i}}^{(0,0)} G_{w\bar{w}j\bar{j}}^{(0,0)}, \\ M_{uu, q} &= D_{u\bar{u}m\bar{m}}^{(0,0)} Q_{u\bar{u}n\bar{n}}^{(0,0)}, M_{ww, q} = D_{w\bar{w}l\bar{l}}^{(0,0)} Q_{w\bar{w}r\bar{r}}^{(0,0)} \end{aligned} \quad (23)$$

且

$$F_{\alpha\bar{\alpha}\beta\bar{\beta}\gamma}^{s, \bar{s}} = \int_{-1}^1 \left\{ \frac{d^s [f_\alpha^1(\xi) P_\delta(\xi)]}{d\xi^s} \frac{d^{\bar{s}} [f_\beta^1(\xi) P_\gamma(\xi)]}{d\xi^{\bar{s}}} \right\} d\xi,$$

$$G_{\alpha\bar{\alpha}\beta\bar{\beta}\gamma}^{s, \bar{s}} = \int_{-1}^1 \left\{ \frac{d^s [f_\alpha^2(\eta) P_\delta(\eta)]}{d\eta^s} \frac{d^{\bar{s}} [f_\beta^2(\eta) P_\gamma(\eta)]}{d\eta^{\bar{s}}} \right\} d\eta,$$

$$D_{\alpha\bar{\alpha}\beta\bar{\beta}\gamma, q}^{s, \bar{s}} = \int_{-1}^1 \left\{ \frac{d^s [\phi_\alpha(\xi_q) P_\delta(\xi_q)]}{d\xi_q^s} \frac{d^{\bar{s}} [\phi_\beta(\xi_q) P_\gamma(\xi_q)]}{d\xi_q^{\bar{s}}} \right\} d\xi_q,$$

$$Q_{\alpha\bar{\alpha}\beta\bar{\beta}\gamma, q}^{s, \bar{s}} = \int_{-1}^1 \left\{ \frac{d^s [f_\alpha^q(\zeta_q) P_\delta(\zeta_q)]}{d\zeta_q^s} \frac{d^{\bar{s}} [f_\beta^q(\zeta_q) P_\gamma(\zeta_q)]}{d\zeta_q^{\bar{s}}} \right\} d\zeta_q \quad (24)$$

式中 $s, \bar{s} = 0, 1, 2; \alpha, \beta = u, w; \delta = i, j, n, m, l, r; \gamma = \bar{i}, \bar{j}, \bar{n}, \bar{m}, \bar{l}, \bar{r}$ 。

综合矩形板和三个子块的刚度矩阵、质量矩阵, 得整个加筋矩形板的振动特征方程为:

$$(K - \Omega^2 M) X = 0 \quad (25)$$

式中 Ω 为含开口裂缝的加筋矩形板的无量纲自振频率。

$$\Omega^2 = \frac{\rho h \omega^2 a^4}{D}, \sigma = \frac{a}{h}, \psi = \frac{H}{h}, \alpha = \frac{B}{b} \quad (26)$$

$$K = \text{diag} \left(4K_p, \frac{6\psi^3\alpha}{\theta_1} K_1, \frac{6\psi^3\alpha}{\theta_2} K_2, \frac{6\psi^3\alpha}{\theta_3} K_3 \right) \quad (27)$$

$$M = \text{diag} \left(\frac{1}{4} M_p, \frac{\beta_1 \gamma_1 \alpha \psi}{4} M_1, \frac{\beta_2 \gamma_2 \alpha \psi}{4} M_2, \frac{\beta_3 \gamma_3 \alpha \psi}{4} M_3 \right) \quad (28)$$

$$X = \{ X_p^T \quad X_1^T \quad X_2^T \quad X_3^T \}^T \quad (29)$$

矩形板与三个子块之间需分别满足位移连续条件,即

$$\begin{aligned} \frac{1}{2} \int_{-\frac{B}{b}}^{\frac{B}{b}} \frac{h}{a} \frac{\partial W(\xi, \eta)}{\partial \xi} d\eta &= \frac{B}{b} U_1(\xi_1, 1), \\ \frac{1}{2} \int_{-\frac{B}{b}}^{\frac{B}{b}} W(\xi, \eta) d\eta &= \frac{B}{b} W_1(\xi_1, 1), \\ U_1(\xi_1, -1) &= U_2(\xi_2, 1), \\ W_1(\xi_1, -1) &= W_2(\xi_2, 1), \\ U_1(\xi_1, -1) &= U_3(\xi_3, 1), \\ W_1(\xi_1, -1) &= W_3(\xi_3, 1) \end{aligned} \quad (30)$$

式中

$$\begin{aligned} \xi &= \xi_1, \quad -1 \leq \xi_1 \leq 1, \\ \xi_2 &= \frac{a_1}{a_2} \xi_1 + \left(\frac{a_1}{a_2} - 1 \right), \quad -1 \leq \xi_1 < 2 \frac{a_2}{a_1} - 1, \\ \xi_3 &= \frac{a_1}{a_3} \xi_1 - \left(\frac{a_1}{a_3} - 1 \right), \quad 2 \frac{a_2}{a_1} - 1 \leq \xi_1 < -1 \end{aligned} \quad (31)$$

考虑到切比雪夫多项式的性质:

$$\begin{aligned} P_n(0) &= \cos\left(\frac{n}{2}\pi\right), P_n(1) = 1, \\ P_n(-1) &= (-1)^{n-1} \end{aligned} \quad (32)$$

及

$$\int_{-1}^1 \frac{P_s(\xi) P_t(\xi)}{\sqrt{1-\xi^2}} d\xi = \begin{cases} 0, & s \neq t \\ \frac{\pi}{2}, & s = t \neq 1 \\ \pi, & s = t = 1 \end{cases} \quad (33)$$

将式(7)和(15)代入式(30),六个等式两边分别同时乘 $P_M(\xi_1)$ ($M=1, 2, 3, \dots, M_1$), $P_L(\xi_1)$ ($L=1, 2, 3, \dots, L_1$), $P_M(\xi_2)$ ($M=1, 2, 3, \dots, M_2$), $P_L(\xi_2)$ ($L=1, 2, 3, \dots, L_2$), $P_M(\xi_3)$ ($M=1, 2, 3, \dots, M_3$) 和 $P_L(\xi_3)$ ($L=1, 2, 3, \dots, L_3$), 并在区间 $(-1, 1)$ 分别对 ξ_1, ξ_2 和 ξ_3 积分,得:

$$\begin{aligned} \pi \sum_{n=1}^{N_1} B_{mn}^1 &= \sum_{i=1}^{I_p} \sum_{j=1}^{J_p} A_{ij}^p \frac{1}{2} \frac{1}{\sigma\alpha} T_{wi,1}, \quad m=1, \\ \pi \sum_{n=1}^{N_1} B_{mn}^1 &= \sum_{i=1}^{I_p} \sum_{j=1}^{J_p} A_{ij}^p \frac{1}{2} \frac{1}{\sigma\alpha} T_{wi,1}, \quad m=2, 3, \dots, M_1, \\ \pi \sum_{r=1}^{R_1} C_{lr}^1 &= \sum_{i=1}^{I_p} \sum_{j=1}^{J_p} A_{ij}^p H_{Li,1}^w \int_{-\frac{B}{b}}^{\frac{B}{b}} f_w^2(\eta_1) P_j(\eta_1) d\eta_1, \quad l=1, \end{aligned}$$

$$\begin{aligned} \frac{\pi}{2} \sum_{r=1}^{R_1} C_{lr}^1 &= \sum_{i=1}^{I_p} \sum_{j=1}^{J_p} A_{ij}^p H_{Li,1}^w \int_{-\frac{B}{b}}^{\frac{B}{b}} f_w^2(\eta_1) P_j(\eta_1) d\eta_1, \\ l &= 2, 3, \dots, L_1 \end{aligned} \quad (34)$$

$$\begin{aligned} \pi \sum_{n=1}^{N_2} B_{mn}^2 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} B_{mn}^1 (-1)^{n-1} H_{mM,2}^u, \quad m=1, \\ \frac{\pi}{2} \sum_{n=1}^{N_2} B_{mn}^2 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} B_{mn}^1 (-1)^{n-1} H_{mM,2}^u, \\ m &= 2, 3, \dots, M_2, \\ \pi \sum_{n=1}^{N_2} C_{lr}^2 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} C_{lr}^1 (-1)^{r-1} H_{lM,2}^w, \quad l=1, \\ \frac{\pi}{2} \sum_{n=1}^{N_2} C_{lr}^2 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} C_{lr}^1 (-1)^{r-1} H_{lM,2}^w, \quad l=2, 3, \dots, L_2, \\ \pi \sum_{n=1}^{N_3} B_{mn}^3 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} B_{mn}^1 (-1)^{n-1} H_{mM,3}^u, \quad m=1, \\ \frac{\pi}{2} \sum_{n=1}^{N_3} B_{mn}^3 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} B_{mn}^1 (-1)^{n-1} H_{mM,3}^u, \\ m &= 2, 3, \dots, M_3, \\ \pi \sum_{n=1}^{N_3} C_{lr}^3 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} C_{lr}^1 (-1)^{r-1} H_{lM,3}^w, \quad l=1, \\ \frac{\pi}{2} \sum_{n=1}^{N_3} C_{lr}^3 &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} C_{lr}^1 (-1)^{r-1} H_{lM,3}^w, \quad l=2, 3, \dots, L_3 \end{aligned} \quad (35)$$

式中

$$T_{wi,1} = \int_{-1}^1 \frac{P_M(\xi_1)}{\sqrt{1-\xi_1^2} \phi_u^1(\xi_1)} \frac{\partial [f_w^1(\xi_1) P_i(\xi_1)]}{\partial \xi_1} d\xi_1 \quad (36)$$

$$H_{SM,g}^G = \int_{-1}^1 \frac{\phi_G^1(\xi_1)}{\phi_G^g(\xi_g) \sqrt{1-\xi_g^2}} P_S(\xi_1) P_M(\xi_g) d\xi_g \quad (37)$$

式中 $S=m, l, i; G=u, w; g=2, 3$ 。

式(25)中列向量 X 中的元素因位移连续性条件并不都是独立的,必须通过式(34)和(35)消去 $(M_1 + L_1 + M_2 + L_2 + M_3 + L_3)$ 个非独立系数。建立独立的列向量 \bar{X} 如下:

$$X = S\bar{X} \quad (38)$$

其中, S 可由式(34)和(35)得出。将式(38)代入式(25),得:

$$(\bar{K} - \Omega^2 \bar{M}) \bar{X} = 0 \quad (39)$$

式中

$$\bar{K} = S^T K S, \bar{M} = S^T M S \quad (40)$$

2 收敛性分析及有限元对比

考虑一裂缝位于加强筋跨中,裂缝位置 $a_2/a=0.5$, 裂缝深度 $H_2/H=0.5$, 板四边简支加强筋两端

也为简支。为便于计算,各个振型函数选取相同的切比雪夫项数。表1给出了不同参数以及不同级数项下的无量纲固有频率 Ω 。

从表1中可知,本文方法具有快速收敛性。在级数项取 30×30 时能够达到三位有效数字精度,

故后文计算中级数项均取为 30×30 。

由式(39)求得频率并得到系数特征向量,分别代入式(7)和(16)可获得矩形板和加强筋的各阶模态。图3为四边简支含开口裂缝的加筋矩形板的前三阶模态图。

表1 不同级数项下四边简支含开口裂缝的加筋矩形板无量纲频率的收敛性

Tab. 1 Convergence of non-dimensional frequency of stiffened rectangular plates simply supported by four sides with open cracks under different series terms

a/h	H/h	b/B	级数项	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
35	1.6	35	10×10	23.9635	51.3603	57.2637	79.2734	96.5071	108.2380
			15×15	23.4157	50.7658	56.1586	78.5681	95.6564	107.6704
			20×20	23.2511	49.5625	55.7563	78.1600	95.5511	107.4475
			25×25	23.2458	49.1586	55.7491	78.1584	95.5457	107.4460
			30×30	23.2434	49.1560	55.7472	78.1579	95.5421	107.4423
50	1.85	50	10×10	30.0684	50.9501	69.8563	80.1057	97.7542	122.5531
			15×15	29.4173	50.4741	69.2003	79.6851	97.2267	122.1702
			20×20	29.2141	50.2768	68.9620	79.4423	96.9106	121.8391
			25×25	29.1493	49.6576	67.9561	79.2351	96.8597	121.7460
			30×30	29.1409	49.6567	67.9534	79.2350	96.8574	121.7446
100	2.45	100	10×10	30.8876	49.4292	72.4951	78.9124	99.6307	127.4462
			15×15	30.8793	49.4180	72.4842	78.8836	99.6145	127.4303
			20×20	30.8767	49.4074	72.4736	78.8708	99.5800	127.4181
			25×25	30.8754	49.3895	72.4698	78.8663	99.5795	127.3782
			30×30	30.8732	49.3854	72.4672	78.8641	99.5784	127.3765

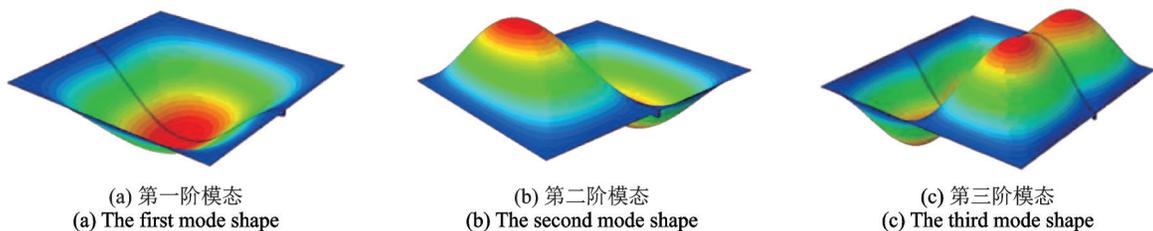


图3 四边简支含开口裂缝的加筋矩形板的前三阶模态图

Fig. 3 The first three modal diagram of stiffened rectangular plates simply supported by four sides with open cracks

为验证方法的正确性,将本文解与有限元解进行对比,如表2所示。加筋矩形板的材料参数为:弹性模量 $E = 68.9 \text{ GPa}$,密度 $\rho = 2670 \text{ kg/m}^3$,泊松比 $\mu = 0.3$,有限元模拟使用ABAQUS软件,板采用壳单元S4R,加强筋采用实体单元C3D8R,共计12050个单元,板四边为简支,加强筋两端也为简支。对比分析了三个不同参数的加筋矩形板,裂缝固定于加强筋的跨中 $a_2/a = 0.5$,裂缝深度为 $H_2/H = 0.2$ 。以有限元结果作为基准,表2显示本文解与有限元解的最大误差只有2.13%,显示出本文结果的可信性。

3 参数分析

分析裂缝深度以及裂缝位置对四边简支含开口裂缝的加筋矩形板振动频率的影响,图4给出了 $a/h = 100, H/h = 2.45, b/B = 100$ 时,不同裂缝位置 a_2/a 下前三阶无量纲固有频率随裂缝深度 H_2/H 的变化。

由图4可知,无量纲频率随裂缝深度增加而减小。对于一阶频率,裂缝越靠近加强筋的中部影响越大;对于二阶频率,因矩形板的变形为反对称,裂缝处于对称轴上,所以裂缝对频率几乎没有影响;对

表 2 本文解与有限元解的对比

Tab. 2 Comparison of present solutions and FE solutions

a/h	H/h	b/B		Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
35	1.6	35	本文解	23.2434	49.1560	55.7472	78.1579	95.5421	107.4423
			有限元解	22.9835	48.8021	55.5587	77.5695	95.0091	106.4480
			误差/%	1.1310	0.7252	0.3393	0.7585	0.5610	0.9341
50	1.85	50	本文解	29.1409	49.6567	67.9534	79.2350	96.8574	121.7446
			有限元解	28.5732	49.2142	67.2195	78.5937	96.4180	121.3334
			误差/%	1.9868	0.8991	0.7549	0.8160	0.4557	0.3389
100	2.45	100	本文解	30.8732	49.3854	72.4672	78.8641	99.5784	127.3765
			有限元解	30.3986	49.3814	70.9561	78.0284	98.5166	126.2241
			误差/%	1.5613	0.0810	2.1296	1.0710	0.1078	0.9130

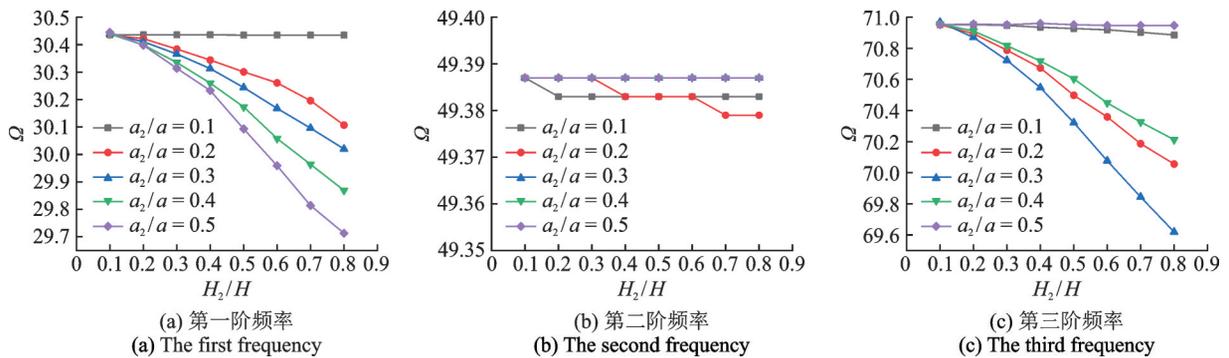


图 4 不同裂缝位置下裂缝深度和位置对前三阶无量纲频率的影响

Fig. 4 Effect of crack depth and location on the first three non-dimensional frequencies under different crack positions

于三阶频率,裂缝越靠近板振型的峰部影响越大。

4 结 论

基于薄板理论和平面应力理论,利用 Chebyshev-Ritz 法建立了裂缝梁加筋矩形板的特征方程。分析了裂缝参数(裂缝位置、裂缝深度)对无量纲频率的影响。计算结果与有限元计算结果对比吻合良好。结论如下:

(1) 裂缝导致四边简支加筋板的各阶固有频率均下降。

(2) 对于四边简支加筋板,裂缝深度以及位置对于频率的影响与板的振型有关。当加强筋裂缝靠近板振型的节线时,裂缝的影响很小;当加强筋裂缝靠近板振型的峰值时,裂缝的影响最大。

(3) 将加强筋沿裂缝分成三个子梁采用平面弹性力学理论分析,与常规采用无质量扭簧近似裂缝处梁的弯曲刚度的方法相比,具有分析简单直接的特点,无需事先确定裂缝的等效刚度。

参考文献:

[1] Rao S R, Sheikh A H, Mukhopadhyay M. Large-amplitude finite element flexural vibration of plates/stiffened

plates [J]. The Journal of the Acoustical Society of America, 1993, 93(6): 3250-3257.

[2] Samanta A, Mukhopadhyay M. Free vibration analysis of stiffened shells by the finite element technique [J]. European Journal of Mechanics-A/Solids, 2004, 23 (1): 159-179.

[3] Peng L X, Liew K M, Kitipornchai S. Buckling and free vibration analyses of stiffened plates using the FS-DT mesh-free method [J]. Journal of Sound & Vibration, 2006, 289(3): 421-449.

[4] Bhaskar K, Pydah A. An elasticity approach for simply-supported isotropic and orthotropic stiffened plates [J]. International Journal of Mechanical Sciences, 2014, 89: 21-30.

[5] 李凯,何书韬,邱永康,等.附加多个集中质量加筋板的自由振动分析[J].中国舰船研究,2015,10(5): 66-70.

Li Kai, He Shutao, Qiu Yongkang, et al. Free vibration analysis of rectangular stiffened plates with several lumped mass [J]. Chinese Journal of Ship Research, 2015, 10(5): 66-70.

[6] 杜菲,马天兵,钱星光,等.基于 Rayleigh-Ritz 法的四边固支加筋板振动研究[J].科学技术与工程,2017,17(11): 137-142.

Du Fei, Ma Tianbing, Qian Xingguang, et al. Vibration research of stiffened plate with four edges clamped

- based on Rayleigh-Ritz method[J]. Science Technology and Engineering, 2017, 17(11): 137-142.
- [7] 游翔宇, 郑文成, 李威, 等. 基于边光滑有限元法的加筋板静力和自由振动分析[J]. 计算力学学报, 2018, 35(1): 28-34.
- You Xiangyu, Zheng Wencheng, Li Wei, et al. Static and free vibration analysis of stiffened plates by ES-FEM using triangular element[J]. Chinese Journal of Computational Mechanics, 2018, 35(1): 28-34.
- [8] 石鹏. 曲型加筋板、壳结构的建模方法与分析研究[D]. 北京: 北京理工大学, 2015.
- Shi Peng. Modeling method and analytical investigation of curved stiffened plate and shell structure[D]. Beijing: Beijing Institute of Technology, 2015.
- [9] Fekrazadeh S, Khaji N. An analytical method for crack detection of Timoshenko beams with multiple open cracks using a test mass[J]. European Journal of Environmental Civil Engineering, 2017, 21(1): 24-41.
- [10] Chondros T G, Dimarogonas A D, Yao J. Vibration of a beam with a breathing crack[J]. Journal of Sound & Vibration, 2001, 239(1): 57-67.
- [11] Rezaee M, Hassannejad R. A new approach to free vibration analysis of a beam with a breathing crack based on mechanical energy balance method[J]. Acta Mechanica Sinica, 2011, 24(2): 185-194.
- [12] Lee J. Identification of a crack in a beam by the boundary element method[J]. Journal of Mechanical Science & Technology, 2010, 24(3): 801-804.
- [13] 蒋杰, 周叮. 两端有裂纹固支深梁的振动特性分析[J]. 建筑结构学报, 2018, 39(增刊2): 183-190.
- Jiang Jie, Zhou Ding. Analysis of vibration characteristics of clamped-clamped deep beams with cracks at ends[J]. Journal of Building Structures, 2018, 39(Sup2): 183-190.
- [14] 蒋杰, 周叮, 胡朝斌. 基于弹性力学的端部有裂缝悬臂梁的自由振动分析[J]. 振动与冲击, 2019, 38(15): 196-201.
- Jiang Jie, Zhou Ding, Hu Chaobin. Free vibration of a cantilever beam with a crack at clamped end based on elasticity theory[J]. Journal of Vibration and Shock, 2019, 38(15): 196-201.
- [15] 孔成, 续秀忠. 含裂纹悬臂梁的振动特性分析[J]. 噪声与振动控制, 2016, 36(5): 31-33.
- Kong Cheng, Xu Xiuzhong. Analysis of vibration characteristics of cracked cantilever beams[J]. Noise and Vibration Control, 2016, 36(5): 31-33.
- [16] Xue J, Wang Y F. Free vibration analysis of a flat stiffened plate with side crack through the Ritz method[J]. Archive of Applied Mechanics, 2019, 89(10): 2089-2102.
- [17] Peng L X, Tao Y P, Liang N, et al. Simulation of a crack in stiffened plates via a meshless formulation and FSDT[J]. International Journal of Mechanical Sciences, 2017, 131-132: 880-893.
- [18] 李国荣, 王磊, 胡朝斌, 等. 典型边界条件下加筋矩形板的横向振动特性分析[J]. 振动与冲击, 2020, 39(16): 261-266.
- Li Guorong, Wang Lei, Hu Chaobin, et al. Analysis of transverse vibration characteristics for stiffened rectangular plates with classical boundary conditions[J]. Journal of Vibration and Shock, 2020, 39(16): 261-266.
- [19] Zhou D, Cheung Y K, Au F T K, et al. Three-dimensional vibration analysis of thick rectangular plates using Chebyshev polynomial and Ritz method[J]. International Journal of Solids & Structures, 2002, 39(26): 6339-6353.

Analysis of transverse vibration characteristics of stiffened rectangular plates with single cracked stiffener

WANG Zhi-qiang, ZHOU Ding, HUO Rui-li, LI Xue-hong

(College of Civil Engineering, Nanjing Tech University, Nanjing 211816, China)

Abstract: The energy method is used to analyze the transverse vibration of simply-supported stiffened rectangular plates with a single crack stiffener. The rectangular plate and the stiffener are cut along the interface, and then the stiffener is divided into several sub blocks along the crack. The energy equations of the transverse vibration of the rectangular plate and each sub blocks are obtained by using the thin plate model and the plane stress theory. The displacement functions of rectangular plates and sub blocks are constructed by using the first kind of Chebyshev polynomials. The transverse vibration characteristic equation of a stiffened rectangular plate with a single cracked stiffener is obtained by the Ritz method and plate-stiffener displacement continuity condition. Finally, the effects of crack depth and crack position on the dimensionless frequency are analyzed.

Key words: stiffened rectangular plates; transverse vibration; crack; Chebyshev-Ritz method

作者简介: 王志强(1995-),男,硕士研究生。电话: 18994086261; E-mail: 530047539@qq.com。

通讯作者: 霍瑞丽(1980-),女,博士,硕士生导师。电话: 025-58139864; E-mail: ruilihuo@njtech.edu.com。